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CONSTRUCTION OF GROUP DIVISIBLE DESIGNS

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SUMMARY

A technique of constructing group divisible incomplete block designs from known designs is described. This method when applied to a special class of designs leads to the solution of one new design and new solutions of some other designs. All solutions have a cyclic structure.

Keywords : Group divisible designs, cyclic structure, merging of treatments.

Introduction

A group divisible design is an arrangement of $v = mn$ treatments in b blocks such that :

(i) each treatment is replicated r times;
(ii) each block contains k distinct treatments, $k < v$;
(iii) the treatments can be divided into m groups of n treatments each, any two treatments being first associates if they belong to the same group and second associates if they belong to different groups;
(iv) two treatments which are mutually ith associates occur together in λ_i blocks, $i = 1, 2$.

Group divisible designs have been extensively tabulated by Clatworthy [1]. Several other new designs have been reported recently by Freeman [3], John and Turner [4] and Dey [2].

In this paper, a method of construction of group divisible designs using a known design is described. This method when applied to a special class of designs leads to the solution of a design hitherto unknown. New solutions of some other designs are also obtained. All the solutions reported have a cyclic structure.

2. The Method

Let there exist a group divisible design D_1 with parameters $v' = m'n'$, b', r', k', $\lambda_1' = 0$, λ_2', m', $n' = st$ ($s \geqslant 2$, $t \geqslant 2$). The association scheme for this design can be written as below:

$$\begin{array}{l} a_{11} \ldots a_{1s}\ a_{1,s+1} \ldots a_{1,2s} \ldots a_{1,(t-1)s} \ldots a_{1,ts} \\ a_{21} \ldots a_{2s}\ a_{2,s} + 1 \ldots a_{2,2s} \ldots a_{2,(t-1)s} \ldots a_{2,ts} \\ \cdot \\ \cdot \\ \cdot \\ a_{m'1} \ldots a_{m's}\ a_{m',s+1} \ldots a_{m',2s} \ldots a_{m',(t-1)s} \ldots a_{m',ts}, \end{array} \tag{2.1}$$

where a_{ij} ($i = 1, \ldots, m'\ j = 1, \ldots, st$) denote the treatments, two treatments being first associates if they belong to the same row of the array (2.1) and second associates otherwise. Replace each of the treatments, $a_{i1}, \ldots, a_{is}$ by a new treatment A_{i1}, each of the treatments $a_{i,s+1}, \ldots, a_{i,2s}$ by A_{i2}, etc., and finally the treatments $a_{i(t-1)s}, \ldots, a_{i,ts}$ by A_{it} for $i = 1, \ldots, m'$. The new array is thus

$$\begin{array}{ll} A_{11} & A_{12} \ldots A_{1t} \\ A_{21} & A_{22} \ldots A_{2t} \\ A_{m'1} & A_{m'2} \ldots A_{m't} \end{array} \tag{2.2}$$

If the same replacement of old treatments a_{ij} by new treatments A_{il} is done in the design D_1, we get a group divisible design, say D_2, with parameters

$$\begin{array}{l} v^* = v'/s = m't,\ b^* = b',\ r^* = sr',\ k^* = k', \\ \lambda_1^* = 0,\ \lambda_2^* = s\lambda_2',\ m^* = m',\ n^* = t. \end{array} \tag{2.3}$$

The association scheme of D_2 is provided by the array (2.2) with two treatments being first associates if they belong to the same row of (2.2) and second associates otherwise.

The procedure described above is equivalent to merging the s treatments belonging to each of the t disjoint sets within the groups. Similar merging can be done when the disjoint sets are of unequal sizes which leads to designs with unequal replications. For details on such designs, the reader is referred to Puri and Nigam [5, 6].

The design D_2 in general has too many replications and designs obtained through this procedure in general may not lead to any useful design. However, if D_1, is taken to be of a special type, new designs can be obtained through this procedure. These are described in the following section.

3. New Designs

Suppose D_1 has $b' = pv'$ blocks ($p \geqslant 1$) *and* a solution of D_1 is obtainable by developing p initial blocks. The design D_2 in such a case has s repetitions of a group divisible design D_3. The parameters of D_3 are

$$v = v'/s, b = b'/s, r = r', k = k', \lambda_1 = 0, \lambda_2 = s\lambda_2', m = m', n = t. \tag{3.1}$$

Further, D_3 also has a cyclic structure. Useful designs obtained thus are given in Table 1.

TABLE 1—PARAMETERS OF THE DESIGNS

Number	v	r	k	b	m	n	λ_1	λ_2	*Number of* D_1
R 55	8	6	3	16	4	2	0	2	*R* 86
R 136	8	10	5	16	4	2	4	6	*R* 86
R 180*a*	24	7	7	24	8	3	0	2	*R* 133
R 197*a*	20	9	9	20	10	2	0	4	*R* 202
R 200*a*	40	9	9	40	10	4	0	2	*R* 202

The design numbers refer to the table of Clatworthy [1].

Initial blocks for each of the designs Table 1 are given in Table 2.

TABLE 2—INITIAL BLOCKS FOR DESIGNS IN TABLE 1

Number	*Initial Blocks*
R 55	(1, 2, 3), (1, 3, 6) mod 8
R 136	(1, 5, 6, 7, 8), (1, 3, 5, 6, 8) mod 8
R 180*a*	(1, 2, 5, 7, 11, 12, 14) mod 24
R 197*a*	(1, 2, 3, 4, 6, 10, 15, 17, 18) mod 20
R 200*a*	(1, 3, 4, 6, 10, 17, 18, 22, 35) mod 40

Solutions for designs *R* 55 and 136 have been reported by Clatworthy [1]. However, his solutions are obtained by duplicating the solutions for designs *R* 54 and *R* 134 respectively. The solutions for these designs given here are without repeated blocks and are non-isomorphic to the ones given by Clatworthy [1].

Design 180*a* has been reported by Freeman [3] who also gives a cyclic solution for this design.

Design 197*a* has been reported by Dey [2]. However, his solution is without a cyclic structure while the design reported here does have a cyclic structure.

Design 200*a* is new as its solution does not appear elsewhere. The average efficiency factor for this design works out to 0.91.

It would be noticed that both the designs *R* 55 and *R* 136 can be obtained by using the design *R* 86 as D_1. However, the design *R* 136 is also complementary to *R* 55. Again, both *R* 197*a* and *R* 200*a* are obtained using the design *R* 202. The initial block for the design *R* 197*a* can also be obtained by reducing mod 20 the elements of the initial block of the design *R* 200*a*.

Another design of interest is the design *R* 57 with parameters $v = 8$, $r = 9$, $k = 3$, $b = 24$, $\lambda_1 = 0$, $\lambda_2 = 3$, $m = 4$, $n = 2$, a solution of which has been reported by Clatworthy [1] by repeating the design *R* 54 three times. A cyclic solution with distinct blocks for *R* 57 can be obtained by developing each of the three initial blocks:

(1, 2, 3), (1, 2, 4), (1, 3, 6) mod 8.

REFERENCES

[1] Clatworthy, W. H. (1973): *Tables of Two-associate Partially Balanced Designs*, Washington D.C. National Bureau of Standards.

[2] Dey, A. (1977). Construction of regular group divisible designs, *Biometrika* **64**: 647-649.

[3] Freeman, G. H. (1976): A cyclic method of constructing regular group divisible incomplete block designs, *Biometrika* **63**: 555-558.

[4] John, J. A. and Turner, G. (1977): Some new group divisible designs, *Jour. Stat. Planning and Inference* **1**: 103-107.

[5] Puri, P. D. and Nigam, A. K. (1975): A note on efficiency balanced designs, *Sankhya B*, **37**: 457-460.

[6] Puri, P. D. and Nigam, A. K. (1978): Merging of treatments in block designs. Submitted.